CHARACTERISTICS OF UNSYMMETRICAL BROADSIDE-COUPLED STRIPS IN AN INHOMOGENEOUS DIELECTRIC MEDIUM

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Abstract

A method is presented for determining the characteristics of unsymmetrical broadside-coupled strips imbedded in an inhomogeneous dielectric medium enclosed by a rectangular shield. The characteristics of the coupled lines have been obtained in terms of even- and odd-mode velocities, coupling coefficients, and even- and odd-mode admittances. Numerical results are given for a variety of configurations.

INTRODUCTION

The purpose of this paper is to evaluate the characteristics of unsymmetrical broadside-coupled strips imbedded in an inhomogeneous dielectric medium. The determination of the properties of the coupled line is necessary for the design of microwave components such as filters, directional couplers, impedance transformers, phase shifters, and hybrid junctions. The parameters needed to characterize the coupled lines are the coupling coefficients, even- and odd-mode characteristic admittances, and the mode phase velocities.

The even- and odd-mode capacitance parameters for coupled symmetrical microstrip lines in an inhomogeneous medium have been studied in detail by Smith [1]. Several authors [2-5] have calculated the characteristics of a pair of broadside-coupled lines on a suspended substrate, but only the case for symmetrical coupled lines has been presented. In this paper, a general method is given for the analysis of the unsymmetrical coupled strips on a suspended substrate enclosed by a rectangular shield. The basic configuration is shown in Fig. 1(a). It can be seen that the dielectric substrate between the coupled lines provides more loading effect for the odd-mode of wave propagation than for the even-mode and results in unequal even- and odd-mode phase velocities. The method for determining the properties of the coupled lines is first to derive a suitable Green's function for the structure. Once the function is obtained, then the self and mutual capacitances of the coupled lines are calculated numerically. Finally, the even-and odd-mode velocities of the coupled lines are determined based on the results derived from the coupled mode theory [6-8]. In this paper, a computer-aided numerical method has been developed to calculate mode velocities and admittances of the coupled lines. The computer program generated can be used as a design tool to obtain circuit configuration for any given even- and odd-mode admittances.

Computed even- and odd-mode velocity and admittance data versus the stripwidth are presented for two sample configurations of a pair of unsymmetrical broadside-coupled lines.

FORMULATION OF THE METHOD

The cross-sectional structure of the coupled lines considered in this paper is shown in Fig. 1(a). Assuming the transverse dimensions of the configuration are relatively small compared to the wavelength, an approximate analysis in terms of a quasi-TEM mode is possible. When a unit potential is applied to the i^{th} strip and the potential is zero on the other strip, the elements of the capacitance matrix of the coupled lines may be written in terms of the unknown charge distribution of the two strips [9-10]. Therefore, the potential $\phi(x,y)$ resulting from the charge density $\sigma_i(x',y')$ on both strips can be expressed, based on the superposition principle, as

$$\phi(x,y) = \sum_{i=1,2} \int_{-w_i}^{w_i} G(x,y;x',y')\, \sigma_i(x',y')dx' \quad (1)$$

The Green's function $G(x,y;x',y')$ for the geometry of Fig. 1(a) is derived by considering a unit line charge located at a point (x',y') in the layer of the dielectric ε_2 region. This function, when b = 1, is then given by

$$G_{1,3}(x,y;x',y') = \sum_{n=1}^{\infty} \frac{B_n}{2A_n} \sin \frac{n\pi}{2w}(x+w)\cdot(1-A_n \tanh \frac{n\pi}{2w}h) \cdot \begin{cases} \sin \frac{n\pi}{2w}(1-y)\cdot(1+A_n \tanh \frac{n\pi}{2w} y') & h \le y \le 1 \\ \sin \frac{n\pi}{2w}(1+y)\cdot(1-A_n \tanh \frac{n\pi}{2w} y') & -1 \le y \le -h \end{cases} \quad (2)$$

$$G_2(x,y;x'y') = \sum_{n=1}^{\infty} \frac{1}{n\pi\bar{\varepsilon}_r A_n} \sin \frac{n\pi}{2w}(x+w) \sin \frac{n\pi}{2w}(x'+w) \cdot \begin{cases} (\cosh \frac{n\pi}{2W} y - A_n \sinh \frac{n\pi}{2w} y)(\cosh \frac{n\pi}{2W} y' + A_n \sinh \frac{n\pi}{2w} y') & y' \le y \le h \\ (\cosh \frac{n\pi}{2W} y + A_n \sinh \frac{n\pi}{2w} y)(\cosh \frac{n\pi}{2W} y' - A_n \sinh \frac{n\pi}{2w} y') & -h \le y \le y' \end{cases} \quad (3)$$

where

$$\bar{\varepsilon}_r = \frac{\varepsilon_2}{\varepsilon_1} = \text{relative dielectric constant of the medium}$$

$$A_n = \frac{1 + \bar{\varepsilon}_r \tanh \frac{n\pi}{2w}(1-h) \tanh \frac{n\pi}{2w} h}{\tanh \frac{n\pi}{2w} h + \bar{\varepsilon}_r \tanh \frac{n\pi}{2w}(1-h)}$$

$$B_n = \frac{2 \sin \frac{n\pi}{2w}(x'+w)}{n\pi\bar{\varepsilon}_r \sinh \frac{n\pi}{2w}(1-h)} \cosh \frac{n\pi}{2w} h \cosh \frac{n\pi}{2w} y'$$

If the stripwidth is divided into M subintervals, then the charge density function $\tau_i(x)$ along the i^{th} strip is assumed to be piecewise linear with M + 1 free parameters. The potential ϕ at the point (x,y) of the i^{th} strip may then be written as

$$\phi_i(x,y) = \sum_{j=1}^{2} \sum_{k=1}^{M+1} \tau_{jk} D_{jk} \qquad i = 1,2 \quad (4)$$

where D_{jk} is a potential at a point (x,y) due to a uniform charge density of magnitude unity on an interval Δx_{jk}.

The charge density function τ_{jk} can be found by solving a set of equations, each of the equations in the form given by (4). The elements of the capacitance matrix are then given by

$$C_{ij} = \sum_{k=1}^{M} \frac{1}{2}(\tau_{jk} + \tau_{jk+1})\Delta x_{jk} \quad i,j = 1,2 \quad (5)$$

It should be noted here that the capacitances C_{11} and C_{12} are found by setting $\phi_1 = 1$ and $\phi_2 = 0$. Likewise, the capacitances C_{21} and C_{22} are found by setting $\phi_1 = 0$ and $\phi_2 = 1$. Fig. 1(b) shows various line capacitances of the unsymmetrical coupled lines. Since the circuit configuration is composed of two unidentical coupled lines embedded in an inhomogeneous medium, the basic definition of even- and odd-mode capacitances is defined with two identical voltages of equal or opposite phase being applied to the lines. However, the currents in the lines may be of different magnitudes. When the inductive parameters of the coupled lines as shown in Fig. 2 are computed, the dielectric inhomogenuity has been disregarded because the dielectric medium is nonmagnetic. Under the condition of two different current magnitudes in the lines, the self and mutual inductances of the coupled lines are found to be

$$L_a = \frac{1}{2}(L_e^a + L_o^a)$$

$$L_b = \frac{1}{2}(L_e^b + L_o^b)$$

$$L_{ab} = \frac{1}{2}\sqrt{(L_e^a - L_o^a)(L_e^b - L_o^b)} \qquad (6)$$

where

$$L_{o,e}^a \; C_{o,e}^a(1) = \mu_o \varepsilon_o$$

$$L_{o,e}^b \; C_{o,e}^b(1) = \mu_o \varepsilon_o$$

$C_e^{a,b}(1)$ = even-mode capacitance of lines a and b with the dielectric layer removed

$C_o^{a,b}(1)$ = odd-mode capacitance of lines a and b with the dielectric layer removed

Note that the results given in (6) agree with those given by Krage and Haddad [6] for the case of two identical coupled lines.

For the unsymmetrical coupled-line system described in this paper, it is found that the theory of coupled transmission lines may be used to determine phase velocities of the even- and odd-mode transmission system where there is distributed coupling between modes.

A computer program has been written utilizing the relationships given above for determining the characteristics of the unsymmetrical coupled strips. The results for two sample configurations are given in the following section.

NUMERICAL RESULTS

The first sample coupled-line configuration that was analyzed is shown in Fig. 3(a), where w/b = 5.0, H/b = 0.2, and $w_1/b = 1.0$. In the computer analysis and results, all physical dimensions have been normalized to the height b of the rectangular shield shown in Fig. 3(a) by setting b = 1 in all computations and with all even- and odd-mode capacitances normalized to the dielectric constant of free space. Capacitance data as functions of the stripwidth of line b for ε_r = 1.0, 2.35, 8.1, and 9.9 resulting from this analysis are presented in Fig. 3(a). It is noted that all capacitances except the even-mode capacitance of line a are rapidly decreased when the stripwidth of line b is reduced. It is also worth mentioning that the capacitance values for $w_2/b = 1.0$ and $\varepsilon_r = 1.0$ have excellent agreement with those given by Kammler [9] for the case of a homogeneous medium. Figures 3(b) and (c) show mode velocities, coupling coefficients, and mode characteristic admittances as functions of the normalized stripwidth of line b for various dielectric media. In this configuration the even- and odd-mode velocity changes versus the normalized stripwidth of line b are very small. The reason for this is simply because the thickness of the dielectric layer is not sufficiently small compared with the ground plane spacing, and therefore a large percentage of the electromagnetic energy concentrates between the two coupling lines. Numerical results for the second sample structure shown in Fig. 4(a), where w/b = 4.0, H/b = 0.1, and w_1/b =0.4, are also given in Figs. 4(a), (b), and (c). In this configuration, when both values of w_1/b and w_2/b are 0.4, the normalized even- and odd-mode velocities have good agreement with the data given by Allen and Estes [5]. Comparing the even- and odd-mode impedances* obtained here with the results given by Allen and Estes [5] or Gish and Graham [4] shows that the values obtained are higher by a factor of 2. Furthermore, using the similar configuration for the homogeneous case, the calculated values of mode impedance using the formulas given here are the same as the calculated results utilizing Cohn's formulas [11] for the case of the side walls removed.† Both results are also higher by a factor of 2 compared with the calculated results using the formulas given in [4] and [5] for the homogeneous case. The cause of this discrepancy is believed to be due to the statement given by Gish and Graham which reads, "the capacitance of the line is just four times that of the reduced line." Actually, the capacitance of the line should be two times that of the reduced line in their case. The same erroneous statement also appears in [5].

CONCLUSIONS

A method has been presented for determining the characteristics of unsymmetrical broadside-coupled strips embedded in an inhomogeneous dielectric medium. The method can also be extended to calculate the characteristics of multiple broadside-coupled strips and offset coupled-strip transmission lines. The admittance parameters of the coupled lines were derived for an inhomogeneous dielectric medium. We believe that the parameters obtained here will be a useful tool in the design of microwave components. Especially, it is believed that the design of unsymmetrical broadside-coupled line directional couplers in the suspended substrate geometry can be worked out by combining the analysis described herein with the analysis procedure given by Cristal [12].

Finally, it is worth mentioning that the method presented has considered only two zero-thickness strips. The effects of finite thickness on the coupled lines can also be easily solved by modifying the Green's function to satisfy the boundary conditions due to finite thickness strips and by utilizing a similar procedure as given by Silvester [13] to solve an integral equation of the charge density distribution around the thick strip surfaces.

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*In this case, for symmetrical coupled lines the even- and odd-mode impedances are simply the respective reciprocals of the even- and odd-mode admittances.

†As w/w_1 increases beyond a value of 5.0 for which Cohn's formulas are valid.

References

1. J. Smith, "The Even and Odd-Mode Capacitance Parameters for Coupled Lines in Suspendes Substrate," IEEE Trans. on MTT, May 1971, pp. 424-431.
2. H. Green and J. Pyle, "The Characteristic Impedance and Velocity Ratio of Dielectric-Supported Stripline," IEEE Trans. on MTT, Jan. 1965, pp. 135-137.
3. J. Dalley, "A Stripline Directional Coupler Utilizing a Non-Homogeneous Dielectric Medium," IEEE Trans. on MTT, Sept. 1969, pp. 706-712.
4. D. Gish and O. Graham, "Characteristic Impedance and Phase Velocity of a Dielectric-Supported Air-Strip

Transmission Line with Side Walls," IEEE Trans. on MTT, March 1970, pp. 131-148.
5. J. Allen and M. Estes, "Broadside-Coupled Strips in a Layered Dielectric Medium," IEEE Trans. on MTT, Oct., 1972, pp. 662-669.
6. M. Krage and G. Haddad, "Characteristics of Coupled Microstrip Transmission Lines, Parts I and II," IEEE Trans. on MTT, April 1970, pp. 217-228.
7. S. E. Miller, "Coupled Wave Theory and Waveguide Applications," B.S.T.J., May 1954, pp. 661-719.
8. W. H. Louisell, "Coupled Mode and Parametric Electronics," New York: Wiley, 1960.
9. D. W. Kammler, "Calculation of Characteristic Admittances and Coupling Coefficients for Strip Transmission Lines," IEEE Trans. on MTT, Nov. 1968, pp. 925-937.
10. R. E. Collin, "Field Theory of Guided Waves," New York: McGraw-Hill, 1960.
11. S. B. Cohn, "Characteristic Impedances of Broadside-Coupled Strip Transmission Lines," IRE Trans. on MTT, Nov. 1960, pp. 633-637.
12. E. Cristal, "Coupled-Transmission-Line Directional Couplers with Coupled Lines of Unequal Characteristic Impedances," IEEE Trans. on MTT, July 1966, pp. 337-346.
13. P. Silvester, "TEM Wave Properties of Microstrip Transmission Lines," Proc. IEE, Jan. 1968, pp. 43-48.

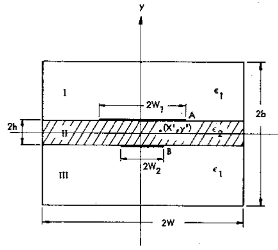
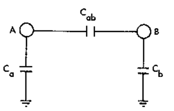


Fig. 1(a). Unsymmetrical Broadside Coupled Strips in a Layered Dielectric Medium



$$[c] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} C_a + C_{ab} & -C_{ab} \\ -C_{ab} & C_b + C_{ab} \end{bmatrix}$$

Fig. 1(b). Electrostatic Capacitive π-Network Model

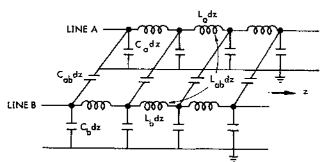


Fig. 2. Equivalent Circuit of Coupled Transmission Lines with Illustration of an Incremental Length

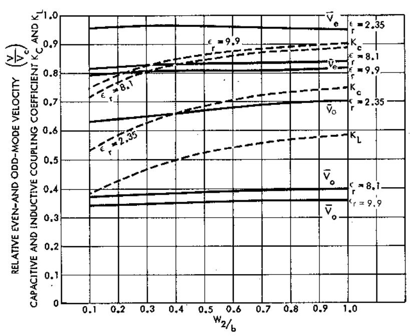


Fig. 3(b). Normalized Odd- and Even-Mode Velocities and Coupling Coefficients as a Function of $(\frac{W_2}{b})$

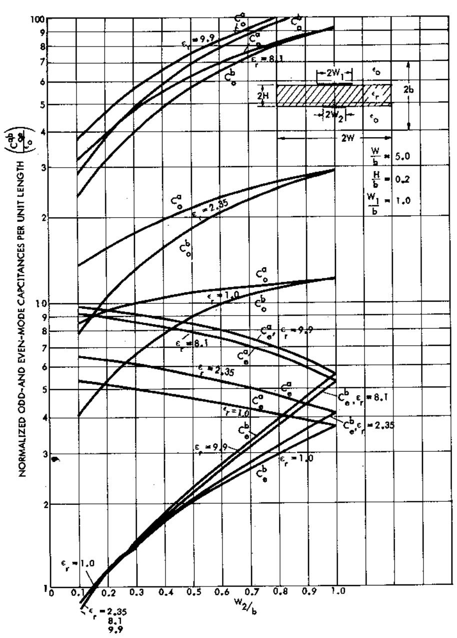


Fig. 3(a). Normalized Odd- and Even-Mode Capacitances as a Function of $(\frac{W_2}{b})$ for Various Dielectric Constants

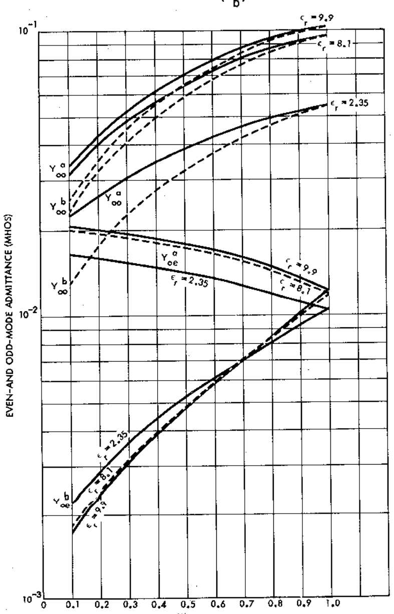


Fig. 3(c). Odd- and Even-Mode Admittances as a Function of $(\frac{W_2}{b})$

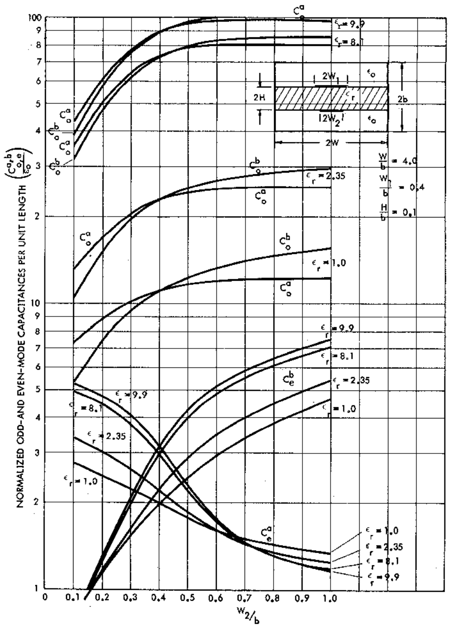


Fig. 4(a). Normalized Even- and Odd-Mode Capacitances as a Function of $(\frac{W_2}{b})$ for Various Dielectric Constants

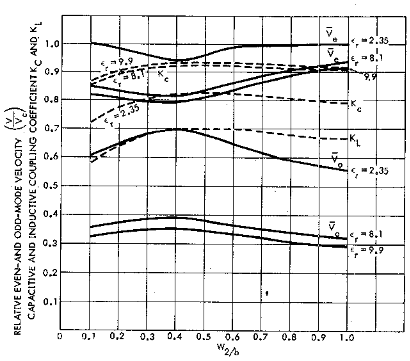


Fig. 4(b). Normalized Odd- and Even-Mode Velocities and Coupling Coefficients as a Function of $(\frac{W_2}{b})$

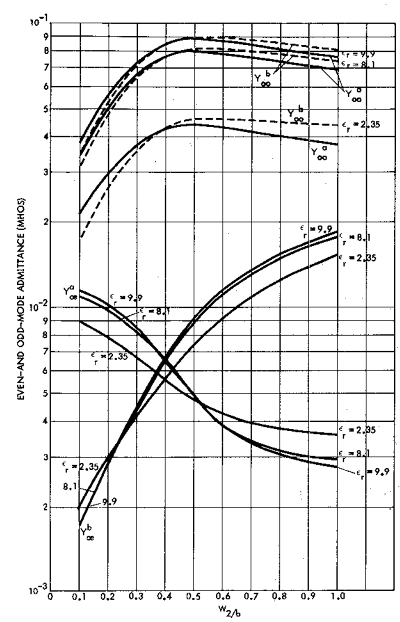


Fig. 4(c). Odd- and Even-Mode Admittances as a Function of $(\frac{W_2}{b})$